Indian Statistical Institute, Bangalore
B. Math (II)
Second Semester 2012-13
Semester Examination : Statistics (II)

Date: 03-05-2013 **Maximum Score 60** **Duration: 3 Hours**

1. A drilling machine is used to make holes in metal sheets. The diameter of the holes drilled using a given shaft may be assumed to be normally distributed with unknown mean θ, $\theta \in \mathbb{R}$ and known variance $\sigma_0^2 > 0$. Let $X_1,\ X_2, ..., X_n$ denote the diameters of n holes made using the given shaft.

   (a) Obtain *minimal sufficient statistic* $T(\mathbf{X})$ *for* θ.

   (b) Suggest an unbiased estimator for θ based on $X_1,\ X_2, ..., X_n$. Does the variance of your estimator attain *Cramér-Rao Lower Bound (CRLB)?* If not, obtain one whose variance attains *CRLB.* Find $I(\theta|\mathbf{X})$, *Fisher Information* in the sample $X_1,\ X_2, ..., X_n$ about the parameter θ. Is it same as $I(\theta|T)$, *Fisher Information* in $T(\mathbf{X})$ about the parameter θ?

   (c) In a more general framework when $\mathbf{X} \backsim f_{\mathbf{X}}(\boldsymbol{x}|\theta)$, are $I(\theta|\mathbf{X})$ and $I(\theta|T)$ same, whenever $T = T(\mathbf{X})$ is sufficient for θ?

[4 + 7 + 7 = 18]

2. Prove that a complete sufficient statistic is independent of every ancillary statistic.

[8]

3. Let $X_1,\ X_2, ..., X_n$ be the random sample from $N(\theta, \sigma^2)$, $\theta \in \mathbb{R}$ and $\sigma^2 > 0$ are both unknown. Derive *size* α *likelihood ratio test* (*LRT*) for the testing problem

$$H_0 : \sigma^2 = \sigma_0^2 \ versus \ H_1 : \sigma^2 \neq \sigma_0^2.$$

where $\sigma_0^2 > 0$ is a specified value. Find p-value. Find 90% *confidence interval for* σ^2.

[7 + 3 + 3 = 13]

[PTO]

4. Let $X_1,\ X_2, ..., X_n$ be a random sample from the distribution with a *pdf* that depends on the unknown parameter $\theta \in (-\infty, \infty)$. Let $T(\mathbf{X}) = T$ be sufficient for θ with *pdf* $f_T(t|\theta) = f(t-\theta),\ t \in (-\infty, \infty);\ \theta \in (-\infty, \infty)$. Here f is a *pdf such that* $f(z) > 0$ on $(-\infty, \infty)$ and $\int_{-\infty}^{\infty} f(z)\, dz = 1$.

   (a) Let us fix $\theta_0, \theta_1 \in (-\infty, \infty)$, $\theta_0 < \theta_1$. Consider the problem of testing of hypothesis

$$H_0 : \theta = \theta_0 \text{ versus } H_1 : \theta = \theta_1. \tag{1}$$

   Let ϕ^* be the test that rejects $H_0 : \theta = \theta_0$ if and only if $T > c$, where $\Pr_{\theta_0}[T > c] = \alpha$. Here $c \in (-\infty, \infty)$ and $\alpha \in (0, 1)$. Show that the test ϕ^* is a level α test for

$$H_0 : \theta \le \theta_0 \text{ versus } H_1 : \theta > \theta_0. \tag{2}$$

   as well.

   (b) If further $\frac{\partial \log f(t)}{\partial t}$ is defined for f and $\frac{\partial \log f(t)}{\partial t}$ is strictly decreasing in $t \in (-\infty, \infty)$; then show that the family $\{f_T(t|\theta) = f(t-\theta) : \theta \in (-\infty, \infty)\}$ has *monotone likelihood ratio property (MLRP)*. Hence or otherwise obtain a *uniformly most powerful (UMP) level α test* for (2).

   (c) How does the test in *(b)* compare with ϕ^* in *(a)*? $[4 + 8 + 2 = 14]$

5. To study the effect of cigarette smoking on platelet aggregation researchers drew blood samples from 11 individuals before and after they smoked a cigarette and measured the percentage of blood platelet aggregation as given in the table below. Platelets are involved in the formation of blood clots, and it is known that the smokers suffer more often from disorders involving blood clots like arterial thrombosis than do nonsmokers. Do the following data support, at $\alpha = 0.05$, the hypothesis that smoking increases platelet aggregation? Find p-value. Find 90% *upper confidence bound* (*UCB*) for the mean increase in platelet aggregation.

Sr.No. $\longrightarrow$ / %$\downarrow$	1	2	3	4	5	6	7	8	9	10	11
Before	25	25	27	44	30	67	53	53	52	60	28
After	27	29	37	56	46	82	57	80	61	59	43

$[8 + 3 + 4 = 15]$